A General Testability Theory: Classes, properties, complexity, and testing reductions

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Outline

1. Motivation
2. Basic framework
3. Testability hierarchy
4. Properties of Class I
5. Elaborated examples
6. Conclusions
Motivation
Testing

Checking the correctness of a system by interacting with it

- In general, tests detect IUT faults
- If a test suite detects the absence of faults, it is complete

Test derivation algorithm

Given a specification and assuming the IUT is of some kind, it derives a good (ideally, complete) test suite

- A different method for each kind of systems
  - LTSs, FSMs, EFSMs, TA, Java, C++, prob. systems, stochastic systems, distributed systems, ...
- Very similar in terms of factors affecting the testing strategy
Our goals

(1) Framework for reasoning about testing independently of the kind of IUT

(2) Classifying testing problems in terms of their hardness ⇒ Difficulty to achieve completeness, the ideal case

(3) Providing general criteria to select finite incomplete test suites when complete ones are not feasible

(4) Exporting test derivation methods between problems
Basic framework
Representing the behavior of a system

A relation between inputs and their sets of possible responses

- Given $I$ and $O$, a function $f : I \rightarrow 2^O$

  \textit{E.g.} if $P$ is a Java program computing the square, its behavior $f$ is such that $f(12) = \{144\}$

- Non-determinism is allowed: $f'(12) = \{144, 145\}$

- $I$ and $O$ can be sequences of other basic symbols

  \textit{E.g.}

  $\begin{array}{c}
  a/0, b/0 \\
  \rightarrow \\
  a/0, b/0 \\
  \rightarrow \\
  b/1
  \end{array}$

- If $f$ represents this FSM then $f(aba) = \{000, 010\}$

- So here $I = \{a, b\}^*$ and $O = \{0, 1\}^*$

- Basic symbols could attach other info: time, prob., etc

  \textit{E.g.} $(a, 4) \cdot (b, 7) \in I$, meaning “give $a$ at time 4, next $b$ at 7”
Candidate IUT behaviors and correct behaviors

- **Basic assumption**: the behavior of the IUT belongs to a given set \( C \) of candidate behaviors (functions)
  
  *E.g.* “we assume that the IUT is an FSM” \( \Rightarrow \) \( C \) consists of all behavior \( f \) we can define with an FSM
  
  *E.g.* “we assume that the IUT is a deterministic FSM” \( \Rightarrow \) only deterministic functions are in \( C \)

- \( C \) implicitly represents our fault model and the hypotheses assumed by the tester about the IUT

- The set of valid behaviors is given by a subset \( E \subseteq C \)
  
  *E.g.* If \( f \) is the spec, \( E = \{ f' \mid f' \preceq f, f \in C \} \)
Distinguishability

- We denote the distinguishability of pairs of outputs from $O$ by a **binary relation** $\not\approx$
  - $o_1 \not\approx o_2$ means that $o_1$ and $o_2$ can be distinguished via observation; otherwise $o_1 \approx o_2$

**E.g.** **Trivial distinguishing relation:** $o_1 \not\approx o_2$ iff $o_1 \neq o_2$

**E.g.** If $st$ denotes system termination and $\perp$ denotes non-termination, we may consider $a \cdot b \cdot st \approx a \cdot \perp$
  - Will $b$ ever come after $a$?? We don’t know!

**E.g.** **Imprecise scale ($\pm 3gr$):**
  - $41gr \approx 43gr$ and $43gr \approx 45gr$ but $41gr \not\approx 45gr$
Complete test suite

Given \((C, E)\), a set of inputs \(I \subseteq I\) is a **complete test suite** if all correct function \(f \in E\) and incorrect function \(f' \in C \setminus E\) are (necessarily) distinguished by some \(i \in I\).

- \(i\) distinguishes \(f\) and \(f'\) if for all \(o \in f(i)\) and \(o' \in f'(i)\) we have \(o \neq o'\).

**E.g.**

<table>
<thead>
<tr>
<th></th>
<th>(i_1)</th>
<th>(i_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f)</td>
<td>{(o_1, o_2}}</td>
<td>{(o_2, o_4}}</td>
</tr>
<tr>
<td>(f')</td>
<td>{(o_2, o_3}}</td>
<td>{(o_1, o_3}}</td>
</tr>
</tbody>
</table>

\(i_1\) does not (necessarily) distinguish \(f\) vs \(f'\), but \(i_2\) does.

\(\{i_2\}\) is a **complete test suite** for \((\{f, f'\}, \{f\})\)
Testability hierarchy
We classify pairs \((C, E)\) into the following classes:

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class I</td>
<td>there is a <strong>finite</strong> complete test suite</td>
</tr>
<tr>
<td>Class II</td>
<td>(next slide)</td>
</tr>
<tr>
<td>Class III</td>
<td>there is a <strong>countable</strong> complete test suite</td>
</tr>
<tr>
<td>Class IV</td>
<td><em>there is a complete test suite</em></td>
</tr>
<tr>
<td>Class V</td>
<td>all pairs ((C, E)) are here</td>
</tr>
</tbody>
</table>
Class II: finitely testable “in the limit”

- **Distinguishing rate (d.r.)** of $\mathcal{I} \subseteq I$: proportion of pairs of correct-incorrect functions in $C$ distinguished by $\mathcal{I}$

- $(C, E)$ is **unboundedly-approachable** if for all $0 \leq \alpha < 1$ we can find some finite $\mathcal{I} \subseteq I$ with distinguishing rate $\geq \alpha$

- Good if $C$ is finite! ...what if $C$ is (countable) infinite?
Let us consider a sequence $C^1 \subseteq C^2 \subseteq \ldots$ of finite sets such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$.

**Class II**

$(C, E) \in \text{Class II}$: there exists a sequence like this such that, for all d.r. $\alpha < 1$, there is a finite set of inputs $\mathcal{I}$ reaching that d.r. for all $C^i$ with $i \geq n$ for some $n$. 
Let $C^1 \subseteq C^2 \subseteq \ldots$ be such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$. 

\[ \begin{array}{ccccccccc}
C & C_1 & C_2 & C_3 & C_4 & \ldots \\
& f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 & f_{10} & \ldots
\end{array} \]
Let $C^1 \subseteq C^2 \subseteq \ldots$ be such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$
Let $C^1 \subseteq C^2 \subseteq \ldots$ be such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$. 

\[C \supseteq C_1 \supseteq C_2 \supseteq C_3 \supseteq C_4 \ldots\] 

\[\text{d.r.} \geq 0.7 \quad \rightarrow \quad i_2, i_7, i_{12}, i_{21}\]
Let $C^1 \subseteq C^2 \subseteq \ldots$ be such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$.
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$d.r. \geq 0.95$ ?

$i2, i7, i12, i21$

$i34, i63, i91$

$d.r. \geq 0.95$ !
Let $C^1 \subseteq C^2 \subseteq \ldots$ be such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$. 

![Diagram showing a sequence of sets $C_i$ and functions $f_i$ with decision reliability (d.r.) ≥ 0.95]
Let $C^1 \subseteq C^2 \subseteq \ldots$ be such that $\bigcup_{i \in \mathbb{N}^+} C^i = C$

And so on for all d.r. $< 1$? Then $(C, E) \in \text{Class II}$
We have

Class I $\subseteq$ Class II $\subseteq$ Class III $\subseteq$ Class IV $\subseteq$ Class V
Examples: Class I (finite testability) and Class II (unboundedly-approachable)

- $C_1$ represents all deterministic FSMs $\{a, b\}^* \rightarrow \{0, 1\}^*$ having at most $n$ states
- $E_1$ represents a single FSM from $C_1$

We have $(C_1, E_1) \in \text{Class I}$

- $\{a, b\}^{2n+1}$ is a complete test suite for $(C_1, E_1)$

Let $C_2 \supset C_1$ represent deterministic FSMs of any size, $f \in C_2$, and $E_2 = \{f\}$. Then, $(C_2, E_2) \notin \text{Class I}$
However, \((C_2, E_2) \in \text{Class II}\). Let us construct \(C^1 \subseteq C^2 \subseteq \ldots\)

- For each way to replace all \(?\) symbols by 0 or 1, take the function representing the smallest FSM behaving like this (or \(f\) if \(f\) does so).
- \(C^1\) is the set of these functions.
However, \((C_2, E_2) \in \text{Class II}\). Let us construct \(C^1 \subseteq C^2 \subseteq \ldots\)

- For each way to replace all ? symbols by 0 or 1, take the function representing the smallest FSM behaving like this (or \(f\) if \(f\) does so).
- \(C^2\) is the set of these functions.
However, \((C_2, E_2) \in \text{Class II}\). Let us construct \(C^1 \subseteq C^2 \subseteq \ldots\)

- For each way to replace all ? symbols by 0 or 1, take the function representing the smallest FSM behaving like this (or \(f\) if \(f\) does so).
- \(C^3\) is the set of these functions.
We have $C^1 \subseteq C^2 \subseteq \ldots$ and $\bigcup_{i \in \mathbb{N}^+} C^i = C_2$.

What is the d.r. of $\{a, b\}^2$ for all $C^i$ with $i \geq 2$?

What is its d.r. for $C^2$?
- We have \( C^1 \subseteq C^2 \subseteq \ldots \) and \( \bigcup_{i \in \mathbb{N}^+} C^i = C_2 \)
- What is the d.r. of \( \{a, b\}^2 \) for all \( C^i \) with \( i \geq 2 \) ?

Only one way of filling the ? symbols (out of \( 2^6 \) possible ways) conforms to \( f \), so the d.r. is \( \geq 1 - \frac{1}{2^6} \).
- We have $C^1 \subseteq C^2 \subseteq \ldots$ and $\bigcup_{i \in \mathbb{N}^+} C^i = C_2$

- What is the d.r. of $\{a, b\}^2$ for all $C^i$ with $i \geq 2$?

What is the d.r. of the same set $\{a, b\}^2$ for $C^3$?
We have $C^1 \subseteq C^2 \subseteq \ldots$ and $\bigcup_{i \in \mathbb{N}^+} C^i = C_2$

What is the d.r. of $\{a, b\}^2$ for all $C^i$ with $i \geq 2$?

There are $2^{14}$ ways to fill the $?$, but the proportion of them behaving as $f$ for $\{a, b\}^2$ is again $\frac{1}{2^6}$. Thus, d.r. $\geq 1 - \frac{1}{2^6}$ again!!!
We have \( C^1 \subseteq C^2 \subseteq \ldots \) and \( \bigcup_{i \in \mathbb{N}^+} C^i = C_2 \).

What is the d.r. of \( \{a, b\}^2 \) for all \( C^i \) with \( i \geq 2 \)?

Hence d.r. of \( \{a, b\}^2 \) is \( \geq 1 - \frac{1}{2^6} \) for all \( C^i \) with \( i \geq 2 \). Same idea for \( \{a, b\}^3 \) (with higher d.r.), etc. These d.r. tends to 1!
We have $C^1 \subseteq C^2 \subseteq \ldots$ and $\bigcup_{i \in \mathbb{N}^+} C^i = C_2$

What is the d.r. of $\{a, b\}^2$ for all $C^i$ with $i \geq 2$?

So, $(C_2, E_2) \in \text{Class II}$
Let $C'_2 \subset C_2$ represent
- some FSM $F$; and
- all FSMs behaving as $F$ for all seq but one of the form $a^k b$
  - one fails for $b$; another fails for $ab$; another for $aab$; etc.

$E'_2$ only represents $F$. Example:

We have $(C'_2, E'_2) \not\in \text{Class II}$

**Intuition:** $n$ tests only distinguish a finite set of pairs (up to $n$)
So \((C_2, E_2) \in \text{Class II}\) but \((C'_2, E'_2) \not\in \text{Class II}\)

But \(C'_2 \subset C_2\), so there are \text{less possible wrong FSMs}\) to worry about in \(C_2\)  (???)

**\(C_2\):** All FSMs are considered, including very faulty ones. Detecting wrong FSMs is easy!

**\(C'_2\):** \(n\) tests distinguish up to \(n\) pairs (out of infinite pairs). Testing \(C'_2\) is not very productive in terms of d.r.!

Testability depends on the \text{hardness} to identify the border correct-incorrect, not on the number of candidates.
Class III (countably testable), Class IV (testable)

Let $C_3$ represent deterministic temporal FSMs:

$s_1 \xrightarrow{i/o} a,b \ s_2$, with $a \leq b$, stands for “if $i$ is received in $s_1$ at time $t \in R$ with $a \leq t \leq b$, then emit $o$ and move to $s_2$”

Let us assume time is continuous

We have $(C_3, E_3) \notin$ Class III, $(C_3, E_3) \in$ Class IV

A fault could have the form $s_1 \xrightarrow{i/o'} a,a \ s_2$ for some $a \in R$. A complete test suite must test all inputs in all real times $t \in R$.
What does it mean to receive \( i \) at time e.g. \( t = \pi \)?

Assuming this strong observability in \( C_3 \) may be artificial.

Let \( C'_3 \) be as \( C_3 \), but we require “\( a < b \)” in \( s_1 \xrightarrow{i/o} a,b \ s_2 \), rather than “\( a \leq b \)”.

We have \((C'_3, E_3) \in \text{Class III}\)

Given \( a, b \in \mathbb{R} \) with \( a < b \), there is a rational \( q \) with \( a < q < b \). Testing all inputs in all rational times (which can be counted) provides completeness!
Examples: \textit{Class V} (all pairs)

- Let $C_5 = \{f, f'\}$, $E_5 = \{f\}$, $I = \{i_1, i_2\}$, $O = \{o_1, o_2, o_3, o_4\}$,

<table>
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<tr>
<th></th>
<th>$i_1$</th>
<th>$i_2$</th>
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<tbody>
<tr>
<td>$f$</td>
<td>${o_1, o_2}$</td>
<td>${o_2, o_3}$</td>
</tr>
<tr>
<td>$f'$</td>
<td>${o_2, o_3}$</td>
<td>${o_1, o_3}$</td>
</tr>
</tbody>
</table>

\((C_5, E_5)\) is only in \textit{Class V}

If the IUT answers $o_2$ to $i_1$, and $o_3$ to $i_2$, the IUT can be $f$ or $f'$

- Non-determinism does not imply non-testability
  - Let $f(i_2) = \{o_2, o_4\} \Rightarrow$ now $\{i_2\}$ is a complete test suite
Class I does not mean finiteness or model simplicity

- Let $C_4$ represent all computable function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $f$ is either strictly increasing or strictly decreasing.
- Let $E_4$ consist of all strictly increasing functions.

$C_4, E_4$ are infinite and cannot be represented by sets of FSMs, but $(C_4, E_4) \in \text{Class I}$ (e.g. $\{5, 6\}$ is complete).

Good testability does not mean simple computation formalisms, but simple correctness-incorrectness border.
Properties of Class I
Properties of Class I

Several simple results about

- Conditions enabling/disabling finite testability

  *E.g.* “C is finite and all pairs are distinguishable $\Rightarrow (C, E) \in \text{Class I}$” (but $\not\Leftarrow$)

- Ways to simplify testing problems

  *E.g.* Equitestability: “A set of functions $C_0$ can be removed from $C$ and $E$ if distinguishing the rest of pairs will necessarily distinguish all functions in $C_0$”

Classifying problems as inside/outside Class I by abstracting most of model-dependant details
Other more elaborated properties:

**Lower bound on the size of a complete test suite**

- Let \( \mathcal{A} \) be a set of disjoint sets of inputs such that, for all set \( B \in \mathcal{A} \), all inputs allowing to distinguish some pair are in \( B \)
- Then, a complete suite needs at least \( |\mathcal{A}| \) inputs
- A pair could be outside \textit{Class I} even if \( |\mathcal{A}| \) is finite

**Alternative characterization of \textit{Class I}**

- A pair is in \textit{Class I} \textit{iff} the set of all sets distinguishing a pair can be partitioned into \( n \in \mathbb{N} \) subsets, in such a way that the intersection of sets in each subset is not null
The case where $C$ is finite

Minimum Complete Suite problem (**MCS**)

Given finite $C, E, \not=, I, O$ extensionally described as arrays, is there a complete test suite with $\leq K \in \mathbb{N}$ inputs?

- **MCS** $\in$ **NP-complete**
  
  Idea: reduction from the Minimum Set Cover to **MCS**

- Measuring **coverage of incomplete test suites**: How good is an incomplete test suite?
  
  - Typically: number of specification *states/transitions/code lines/etc* reached by tests
  
  - A model-independent choice: **Least required hypothesis**
Assuming an hypothesis $\equiv$ removing elements from $C$

- "The IUT is deterministic" $\Rightarrow$ all non-deterministic functions removed from $C$

An incomplete test suite may become complete after removing functions

Given $\mathcal{I} \subseteq I$, its least completeness hypothesis is the least hypothesis (i.e. function removal) making $\mathcal{I}$ complete

- How good is $\mathcal{I}$? How far is it from being complete?

- We prefer those incomplete test suites being closer to be complete $\Rightarrow$ smaller least completeness hypothesis
Let $C, E, \emptyset, I, O$ and $\mathcal{I} \subseteq I$ be finite sets.

**Minimum Function Removal problem (MFR)**

Can $\mathcal{I}$ become complete by removing $\leq K$ functions from $C$?

**Minimum Function Removal via Hypotheses problem (MFH)**

Given hypotheses $\mathcal{H}_1, \ldots, \mathcal{H}_n \subseteq C$, can $\mathcal{I}$ become complete by assuming some hypotheses removing $\leq K$ functions from $C$?

**Minimum Hypotheses Assumption problem (MHA)**

Given hypotheses $\mathcal{H}_1, \ldots, \mathcal{H}_n \subseteq C$, can $\mathcal{I}$ become complete by assuming $\leq K$ hypotheses?

$\textbf{MFR} \in \mathcal{P}$ (complexity $O(|C|^{5/2} + |C|^2 \cdot |I| \cdot |O|^2)$)  
($\textbf{MFR}$ reduced to Minimum Vertex Cover in bipartite graphs)

$\textbf{MFH} \in \text{NP-complete}$ ($3$-SAT reduced to $\textbf{MFH}$)

$\textbf{MHA} \in \text{NP-complete}$ (Minimum Set Cover reduced to $\textbf{MHA}$)
Reducing testing problems to other testing problems

**Testing problem**: set of pairs \((C, E)\) for the same \(I\)

**E.g.** If \(C\) represents all deterministic FSMs and \(E_i\) represents the \(i\)-th deterministic FSM (used as specification) then
\[
\mathcal{T} = \{(C, E_1), (C, E_2), \ldots\} \equiv \text{“testing det. FSMs”}
\]

**Testing reduction**: \(\mathcal{T}_1 \leq_F \mathcal{T}_2\)
If $\mathcal{T}_1 \leq_F \mathcal{T}_2$ and there is a method for deriving complete test suites for $\mathcal{T}_2$, then there is so for $\mathcal{T}_1$

- $\leq_F$ is a preorder
- If $\mathcal{T}_1 \leq_F \mathcal{T}_2$ then if $\mathcal{T}_2 \in \text{Class I}$ then $\mathcal{T}_1 \in \text{Class I}$

How do we prove $\mathcal{T}_1 \leq_F \mathcal{T}_2$?

- By mapping computation formalisms into each other
  
  \textit{E.g.} We prove that formalisms of $\mathcal{T}_2$ simulate formalisms of $\mathcal{T}_1$

- By mapping the correctness-incorrectness border
  
  \textit{E.g.} We prove that $\mathcal{T}_2$ keeps the form of faults as $\mathcal{T}_1$ even though formalisms of $\mathcal{T}_2$ cannot simulate those of $\mathcal{T}_1$
Examples of $\leq_F$

**E.g.**  
\[ \mathcal{T}_1 \equiv \text{“Testing det. FSMs with } \leq n \text{ states”} \]
\[ \mathcal{T}_2 \equiv \text{“Testing det. EFSMs with } \leq m \text{ states where variables take up to } v \text{ values”} \]
\[ \mathcal{T}_3 \equiv \text{“Testing det. TEFSMs (discrete time) with } \leq p \text{ states up to time } k” \]

- For some $n, m, v, p, k$ we have $\mathcal{T}_3 \leq_F \mathcal{T}_2 \leq_F \mathcal{T}_1$
- Idea: mapping formalisms into equivalent ones
In general, $\leq_F$ does not require mapping formalisms into formalisms but mapping the correctness-incorrectness border.

**E.g.** Turing Machines and Finite Automata $\{0, 1\}^* \rightarrow \{\text{yes, no}\}$

$\mathcal{T}_1 \equiv \text{“Testing terminating Turing Machines of any size where wrong TMs answer right for at most k inputs”}$

$\mathcal{T}_2 \equiv \text{“Testing Finite Automata of any size where wrong FA answer right for at most k inputs”}$

- **FA cannot simulate** TMs, but $\mathcal{T}_1 \leq_F \mathcal{T}_2$
  - $e : \mathcal{T}_1 \rightarrow \mathcal{T}_2$ returns the same pair $p_2 \in \mathcal{T}_2$ for all $p_1 \in \mathcal{T}_1$
  - $t : 2^I_2 \rightarrow 2^I$ is the identity function
  - Idea: Any complete test suite for $\mathcal{T}_2$ must consist of at least $k + 1$ inputs $\Rightarrow$ this is also complete for $\mathcal{T}_1$
Elaborated examples
Hennessy’s semantic framework

- Modeling Hennessy’s processes framework into ours
  - Testing semantics: processes are compared in terms of what tests are passed/failed
  - Must testing semantics: all possible executions of the process must pass the test
  - Our functions denote the acceptance sets after each trace

  **E.g.** \( f(abc) = \{\{a\}, \{b\}, \{a, b\}\} \) and \( f'(abc) = \{\{a, b\}\} \) are must-distinguished by input (test) \( abc \) (\( f \) may not accept \( a \) after \( abc \), whereas \( f' \) always does)

- Must semantics: we consider \( \{\{a\}, \{b\}, \{a, b\}\} \in O \), \( \{a, b\}\) \( \in O \) and \( \{\{a\}, \{b\}, \{a, b\}\} \neq \{\{a, b\}\} \)

- By using the previous tree trick and other calculations \( \Rightarrow \) This problem \( \in \text{Class II} \)
Testing FSMs with rational timeouts

- Modeling FSMs with rational timeouts ($0 < t \leq M$)
  - It is known testing $TA \in \text{Class I}$ if we know the number of states and upper/lower bounds of intervals are integers
    $\Rightarrow$ time can be discretized into a finite set of regions
  - If FSMs timeouts can take any rational value within a dense interval (e.g. arbitrarily close to 0)
    $\Rightarrow$ Discretizing is not possible $\Rightarrow \not\in \text{Class I}$

- Our problem is in $\text{Class II}$
  - $C^1 \subseteq C^2 \subseteq \ldots$: each $C^i$ is built from all trees of depth $i$
    where all timeouts are multiple of $\frac{1}{i!}$
  - Test cases may miss states where we stay a short time
  - Still, the lost d.r. tends to 0 $\Rightarrow \text{Class II}$
Testing increasing magnitudes machines

- We study several kinds of FSMs where transitions can also depend on an increasing magnitude.
- All of them are reduced to testing FSMs.
  - Some reductions relate problems where one denotes infinite sets $C$ and the other finite sets $C$.
  - Some reductions relate problems where one has an infinite continuous magnitude domain and the other has a finite magnitude domain.
  - **Problem decomposition**: A series of $n$ reductions proves that the problem of dealing with $n$ magnitudes can be reduced to the problem of using only one.
Conclusions
Conclusions

- **General framework** for reasoning about testing
- **Classification** of problems in terms of the difficulty to achieve completeness
- **Conditions** enabling/disabling/preserving testability
- Measuring “how complete” is an **incomplete** test suite
- **Exporting** methods from a testing problem to another
Future work

- Investigating current and new (sub-)classes
  - **Class I**: If representing \((C, E)\) takes size \(n\), what is the size of complete test suites w.r.t. \(n\)? polynomial? exponential?
  - **Class II**: How fast does d.r. tend to 1 as more test cases are added? **Marginal utility**: After applying \(n\) tests, what is the gained d.r. after applying one more? When should we stop testing?
  - **Adaptive testing**, i.e. the test suite is not set in advance but depends on previous tests responses

  _E.g._ \(T \equiv \text{“testing if } f \text{ fulfills the condition } f(f(3))=4\”\)
  - \(T \not\in \text{Class I: I don’t know } f(3)\), so an a priori test suite must test _all_ numbers
  - If testing is adaptive: \(\{3, f(3)\}\) is complete \(\Rightarrow T \in \text{AdaptiveClass I}\)

- Relating this theory with **learnability theory**
Thank you for your attention!