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Functions and Lazy Evaluation in Prolog

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Abstract

There are several proposals for extending Prolog with functional capabilities. The basic idea is to enlarge the language with function definitions that are translated (or expanded) into Prolog predicates, analogously to what is done for *Definite Clause Grammars* (DCG's). It is easy to perform such a translation for a basic functional extension, but it requires an additional effort to incorporate more sophisticated functional capabilities such as higher order, lambda abstractions and lazy evaluation. In this paper we describe an extension that covers all these features. The main novelty is our treatment of laziness, as it is (optionally) associated to data type constructors instead of functions. We have found this approach very flexible, easy to use and efficient.

Keywords: Functional programming, logic programming, lazy evaluation.

1 Introduction

Both functional and logic programming styles provide appealing resources for programming and there have been different approaches for combining both paradigms in order to amalgamate the most interesting capabilities of them. This is the core idea of functional-logic languages such as Curry [8,9] or Toy [4,13]. They have a strong influence of Haskell in their syntax, higher order capabilities, type inference systems, etc. Moreover, following Haskell, lazy evaluation has been incorporated as a primitive feature. In a certain sense (apart from type classes and some other sophisticated features) they can be seen as an extension of Haskell. In fact, a big collection of Haskell programs runs in Toy or Curry with minor changes. In addition, these functional-logic languages incorporate from logic programming non-deterministic functions, logic variables treated by a generalization of unification (*narrowing*) and constraint solvers. This approach has been successfully developed both from the theoretical and practical points of view.

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Nevertheless, one could think about the integration of functional and logic programming from another perspective, starting from a logic programming language as the host language, and then extending it with functional capabilities. This approach has also been investigated and developed in systems like Mercury [15]. Another possible alternative is to take standard Prolog as the base language and add functional capabilities to it, by translating function rules into Prolog predicates, analogously to what is done with DCG's. Then, programs would be a mixture of standard Prolog predicates and function definitions in such a way that functions can be used in predicate definitions and predicates can be used as conditions in function rules. The idea is not new and it is not difficult to implement a basic extension covering first order, eager functions. But things get more complex when we also want higher order functions and laziness.

In this paper we are interested in this last approach which has been explored in [5,1]. The main novelty of our proposal is the way we deal with laziness, as we deal with lazy data structures instead of lazy functions. In fact, every function is eager in our setting. This is close to the form in which lazy evaluation is expressed in eager functional languages such as OCaml [11] or Scheme, in which *lazy thunks* are used to suspend the evaluation of expressions. These suspended expressions can be explicitly forced to evaluate later on in such a way that this evaluation is ensured to be performed only once because the value of the first evaluation is cached to be shared in future invocations. These *lazy thunks* can be easily simulated in functional Prolog. To suspend an expression e we just replace it with $\lambda x.e$, using a variable x not present in e . We can enforce the evaluation of a suspended expression by just applying it to a *dummy* argument. This mechanism will be the core of our approach to laziness, but still some disturbing issues must be faced in order to achieve our goals:

- delaying and forcing the evaluation of an expression must be explicitly encoded by the programmer. But then he must worry about the appropriate points for introducing suspensions and for evaluating them, the types (if required by the language) of suspended expressions, etc. Programs become more difficult to encode and less readable when all this information is explicitly encoded;
- in our context, with non-deterministic computations involved, these delayed computations may destroy our intended semantics for programs. We do not provide a formal semantics in this work, but our informal argument is as follows: lazy declarations can achieve termination for functions that diverge under eager evaluation. But assuming termination we want to obtain the same result(s) for a function call, with eager or lazy evaluation, or even with a mixture of both. This corresponds to the *call-time* choice semantics adopted in functional-logic systems like Toy or Curry. The above mechanism for lazy evaluation does not satisfy our aim, and we need to introduce some *sharing structure* (as in Toy and Curry).

We have found an easy and satisfactory solution for both problems: the user must only declare the data constructors wanted to be lazy and the translation will do the work of manipulating delayed computations, that are transparent to the programmer. On the other hand, we manipulate an extended version of suspensions

that incorporate sharing⁴.

The paper is organized as follows: Section 2 starts by introducing a suitable syntax for functions in Prolog and a basic (one-to-one) translation of function rules into predicate clauses (we are not looking for a sophisticated evaluation mechanism like in functional-logic systems). Section 3 introduces higher order capabilities and lambda abstractions. Section 4 is devoted to explore some alternatives for lazy evaluation. In Section 5 we give a short idea about the efficiency of this approach. Finally Section 7 presents some conclusions and future directions.

Our extension does not use any special feature of a concrete Prolog system. It has been developed under SWI-prolog but it has also been tested under Sicstus.

2 Function rules as Prolog clauses

A function rule for a function f is written as:

$$\underbrace{f(A_1, \dots, A_n)}_{\text{head}} = \underbrace{Exp}_{\text{body}} :- \underbrace{C_1, \dots, C_m}_{\text{conditions}}$$

where the A_i 's are the arguments of the function, Exp is any expression possibly including some function calls and C_1, \dots, C_m is an optional sequence of conditions with the same syntax and meaning as in Prolog clauses, but possibly containing function calls without any special restriction. For example, the factorial function can be defined as:

```
fact(0) = 1.
fact(N) = N*fact(N-1) :- N>0.
```

These rules are transformed into standard Prolog clauses by adding an argument in the head for getting the result of evaluation and flattening the function calls of the body. The corresponding clauses for *fact* are:

```
fact(0,1).
fact(N,R) :- N>0, -(N,1,N1), fact(N1,R1), *(N,R1,R).
```

In the body of the second rule we flatten the expression $N * fact(N - 1)$ into the corresponding predicate calls. Arithmetical functions and other commonly used functions are all defined in a separate file *prelude* that we can import in our program with the declaration `:- import(prelude)`. In particular, this file contains the following definitions (that are used in our function *fact*):

```
-(X,Y) = Z :- Z is X-Y.
*(X,Y) = Z :- Z is X*Y.
```

Now, for evaluating *fact(3)* we must solve the goal⁵ *fact(3,R)* and we will obtain the result as the Prolog answer $R=6$.

⁴ As an experimental feature, the translator incorporates the option of disabling sharing, obtaining *run-time* choice.

⁵ Prolog prompt goals are not translated and the user must call functions with an additional argument for obtaining the desired result. This could be changed in the future, expanding these goals like conditions of functions/predicates.

There is an important aspect about the meaning of rules that we must clarify. Given a function rule, the order in which computation is performed is the following: firstly parameter passing is done by unification, then conditions are satisfied and finally the body of the rule is evaluated to obtain the result. This is not the only possible translation. An alternative way to proceed could be for example to do parameter passing, evaluate the body of the function and then satisfy the conditions.

<i>% function definition</i>	<i>% proposed translation</i>	<i>% alternative translation</i>
$f(a) = 0 :- !.$	$f(a,R) :- !, R=0.$	$f(a,R) :- R=0, !.$
$f(b) = 1.$	$f(b,1).$	$f(b,1).$
$g = loop :- h(1).$	$g(R) :- h(1), loop(R).$	$g(R) :- loop(R), h(1).$
$g = 3.$	$g(3).$	$g(3).$
$loop = loop.$	$loop(R) :- loop(R).$	$loop(R) :- loop(R).$
$h(0).$	$h(0).$	$h(0).$

Fig. 1. Alternative translations.

The difference becomes relevant in some situations. For example, consider the function f and the corresponding translations of Figure 1. The call $f(Y) = 1$ corresponds to the goal $f(Y,1)$ for which there is no answer with the proposed translation: by the first rule Y is unified with a , the cut discards the other rule, and then the unification of the body $1 = 0$ fails. If we use the alternative translation, this goal would fail by the first rule, but it would succeed by the second one with the answer $Y=b$. Something similar happens with the function g , that returns 3 with the proposed translation, but loops with the second.

We prefer the first translation as it matches our intended meaning for functions, but in absence of a formal semantics we can not claim that any of both translations is correct or wrong. The important point here is to fix a clear behaviour for functions and this is done by defining the sequence of computations. On the other hand, we can get the effect of the other alternative for the function f of Figure 1 defining f as:

```
f(a) = R :- !, R=0.
f(b) = 1.
```

and vice versa: the proposed translation can also be simulated using the alternative one.

We have shown the way in which function rules are translated. Predicates are translated in a direct way, flattening functions calls in the conditions of clauses. For example:

```
doubleFact(N,M) :- fact(N)=F, M=2*F.
```

is translated into:

```
doubleFact(N,M) :- fact(N,F1), F1=F, *(2,F,M1), M=M1.
```

Another interesting thing when mixing functions and predicates may be to use a predicate as a function choosing one or some arguments of the predicate as the returning value(s) of the function. The file *prelude* provides a projection function

proj for this purpose. This projection is useful in general to exploit the built-in predicates of Prolog in a functional style. A call of the form *proj*(*T,G*) evaluates the goal *G* and returns the Prolog term *T* with the corresponding bindings for variables. For example, the following function takes a Prolog term and returns a pair with the main functor and the arity of it:

```
fun_ar(Term) = proj((Name,Arity), functor(Term,Name,Arity)).
```

The call *fun_ar*(*a(b,X)*) returns the pair (*a,2*).

3 Higher order functions and λ -abstractions

For higher order functions we perform a translation into first order using a well known technique that was also implemented in the language Toy [4]. The idea is to consider every partial application as a Prolog term instead of a proper function call and to use a new function @ (read *apply*) for applying these partial applications to arguments. The function @ is defined as an infix operator and the translation generates its rules for (every partial application of) all the functions of the program. As an example, consider the program:

```
map(F, []) = [].
map(F, [X|Xs]) = [F@X|map(F,Xs)].
```

```
neg(L) = map(not,L).
```

```
not(true) = false.
not(false) = true.
```

Notice the explicit use of the function @ in the second rule. The translation produces:

```
map(F, [], R) :- R=[].
map(F, [X|Xs], R) :- @(F,X,Fx), ap(F,Xs,Fxs), R=[Fx|Fxs].
```

```
neg(L,R) :- map(not,L,L1), R=L1.
```

```
not(true,R) :- R=false.
not(false,R) :- R=true.
```

together with the rules for @ corresponding to partial applications of all the functions of the program:

```
@(map,F,map(F)).
@(map(F),L,L1) :- map(F,L,L1).
@(neg,L,L1) :- neg(L,L1).
@(not,X,Y) :- not(X,Y).
```

Figure 2 shows the resolution steps for evaluating *neg([true,false,false],L)*, i.e., the goal *neg([true,false,false],L)* using this translation. The clause used in each step is annotated on the right. The step marked with (@₄) evaluates the partial application *not* on the argument *true*, obtaining the (total) application *not(true,Fx)*. And the

$neg([true,false,false],L)$	\rightsquigarrow	(neg)
$map(not,[true,false,false],L1), L=L1$	\rightsquigarrow	(map_2)
$@(not,true,Fx), map(not,[false,false],Fxs), L1=[Fx Fxs], L=L1$	\rightsquigarrow	$(@_4)$
$not(true,Fx), map(not,[false,false],Fxs), L1=[Fx Fxs], L=L1$	\rightsquigarrow	(not_1)
$Fx=false, map(not,[false,false],Fxs), L1=[Fx Fxs], L=L1$	\rightsquigarrow	$(=)$
$map(not,[false,false],Fxs), L1=[false Fxs], L=L1$	\rightsquigarrow	(map_1)
\dots		
$L=[false,true,true]$		

Fig. 2. Higher order goal example

next step (marked with (not_1)) evaluates this call obtaining $Fx=false$. The rest of steps proceed in a similar way until getting the expected answer $L=[false,true,true]$.

The system also allows λ -abstractions or anonymous functions of the form $\lambda x.e$ using the syntax $fun(x,e)$. In general, a generalized abstraction of the form $\lambda x_1 \dots x_n . e$ can be written as:

$$fun(\underbrace{x_1, \dots, x_n}_{arguments}, \underbrace{e}_{body})$$

In a first step, the translation introduces a new function $funN$ for each anonymous function (where N is a counter of λ -abstractions of the program). The function $funN$ is defined according to the arguments and body of the λ -abstraction and the abstraction itself is replaced by a call to $funN$. For example, the function:

$$f(L) = map(fun(X,X*2),L).$$

will be translated in this first step to:

$$\begin{aligned} f(L) &= map(fun1,L). \\ fun1(X) &= X*2. \end{aligned}$$

In a second step these functions are translated in the usual way. Notice that in general the body of a λ -abstraction can use variables that are not arguments of the abstraction itself, but come from the external context. For example, we can define:

$$g(X) = fun(Y,X+Y).$$

Here X is not an argument of the abstraction but it appears in the body. Every argument of this kind must be passed to the function $funN$, so the first step would produce in this case:

$$\begin{aligned} g(X) &= fun2(X). \\ fun2(X,Y) &= X+Y. \end{aligned}$$

Here $fun2(X)$ is a partial application that expects an argument Y for evaluating. The system checks the arities of the functions and generates the appropriate apply's for them.

4 Lazy evaluation in strict languages

The question about laziness versus strictness is an old debate in the functional community. From a theoretical point of view, lazy evaluation may be preferred to eager evaluation, but from a practical point of view both alternatives have good arguments to support them. A detailed discussion on this topic is beyond the aim of this paper, but we can point out that eager evaluation is usually considered to be more efficient than lazy evaluation in general, while lazy evaluation is a very nice resource for the programmer to encode elegant solutions for problems that involve *infinite* (or *incomplete*) *data structures*.

We adopt a middle way: our functions are strict by default, but we allow lazy evaluation by means of explicit annotations. This idea is not new as the functional extension of Prolog proposed in [5] allows lazy annotations for functions. Furthermore, some strict functional languages such as OCaml provide resources to suspend the evaluation of expressions and to force its evaluation when needed. In the next Sections we explore these approaches.

4.1 Lazy annotations on functions

The proposal of [5] allows to declare a function to be evaluated lazily. In the translation to Prolog predicates, the evaluation of those functions is suspended until the result is demanded by unification at some predicate. This is done by means of the primitive control *freeze(Var,Goal)* that delays the execution of *Goal* until *Var* is bound. To illustrate this translation consider the following program:

```

from(N) = [N|from(N+1)].

take(0,L) = [] :- !.
take(N,[]) = [] :- !.
take(N,[X|Xs]) = [X|take(N-1,Xs)].

firstNats(M) = take(M,from(1)).

```

If these functions are translated as explained before it is easy to understand that the evaluation of the function *firstNats(3)* will not terminate due to the non-termination of *from*. But declaring the function *from* to be lazy allows to make a different translation for it (the translation for *take* and *firstNats* does not change):

```

from(N,L):- freeze(L,fromLazy(N,L)).
fromLazy(N,[N|L]):- +(N,1,N1), from(N1,L).

```

Now consider again the evaluation of the function *firstNats(3)*, i.e., the goal *firstNats(3,L)*. The goal *from(1,L1)* generates a suspended call to *fromLazy(1,L1)*. The variable *L1* will be demanded by unification when evaluating *take(3,L1,L)*. Then we obtain a list with head 1 followed by a suspended call to *fromLazy(2,L2)*. In this way we obtain a list with the elements 1,2 and 3, followed by a suspended call to *take(0, <susp>, K)* that provides the answer $K = []$ using the first clause of *take*.

The translation is simple and works as expected, but it is difficult in general to select the set of functions that must be declared as lazy in a program. A non appropriate selection can produce non-termination or an inefficient translation. Consider the classic sieve of Eratosthenes for generating the (infinite) list of primes, programmed as follows:

```
primes = sieve(from(2)).

sieve([X|Xs]) = [X | sieve(filter(fun(Y,mod(Y,X)=\=0), Xs))].

firstPrimes(N) = take(N,primes).
```

The functions *from* and *take* are defined as in the previous example. The function *filter*(*P*,*L*) removes from the list *L* those elements that do not satisfy the property *P* (*P* must be a boolean function). Then *filter*(*fun*(*Y*,*mod*(*Y*,*X*)=*\=0*),*Xs*) removes the multiples of *X* from *Xs*. Finally, *firstPrimes*(*N*) returns the first *N* prime numbers.

If we make no lazy declaration, every call to *firstPrimes* will loop because *from* produces an infinite list, so we should declare *from* as a lazy function. But the program still loops, because *filter* will demand the evaluation of the (complete) list. Then we should also declare *filter* as a lazy function. The program still loops because *sieve* recursively requires the evaluation of the list. A solution could be to make lazy annotations for every function in the program, thus getting a completely lazy program. This works, but the program would be quite inefficient. In fact, for this particular case, the program will work just declaring the functions *from*, *filter* and *sieve* to be lazily evaluated. This short example shows that it is not easy to locate the appropriate functions that must be lazily evaluated, and the problem would become very hard with large programs.

Notice that in the previous examples what is actually needed to be lazy are lists as a data type, more than a concrete set of functions of the program. This idea will guide the treatment of laziness that we propose.

4.2 Lazy annotations on data type constructors

Some (strict) functional languages provide the possibility of making explicit annotations for *deferred computations*. For example, OCaml [11] allows to suspend the evaluation of an expression *e* by creating a *lazy thunk* of the form:

$$\text{let } \textit{thunk} = \textit{lazy } e;;$$

The type of this thunk is *'a Lazy.t* where *a* is the type of *e*. Later this expression can be explicitly forced to evaluate by writing:

$$\textit{Lazy.force } \textit{thunk};;$$

This returns the actual value of *e* (of type *a*) and stores it for avoiding reevaluation. Using this feature we can declare lazy data types. For example, the type *lList* of lazy lists of elements of type *a* can be declared as the empty list *Nil* or a pair representing the head (of type *a*) and the rest of the list (that is a lazy list of elements of type *a*):

```
type 'a listL = Nil | Cons of ('a * (('a listL) Lazy.t));;
```

With this type we can define an infinite list and some additional functions for testing it:

```
let rec from n = Cons (n, lazy (from (n+1)));;

let rec take n lst = match n, lst with
  | _, Nil -> Nil
  | 0, _ -> Nil
  | _, Cons (x, xs) -> Cons (x, lazy (take (n-1) (Lazy.force xs)));;

let rec nf l = match l with
  Nil -> []
  | Cons (x, xs) -> x :: nf (Lazy.force xs);;
```

The functions *from* and *take* have the same meaning as in previous examples. Notice that they must explicitly suspend and force evaluation according to the declared type. Finally, we can force the evaluation of a lazy list using the function *nf*, which takes a lazy list and returns a standard list in normal form (without any deferred computation). For example, the expression *nf (take 4 (from 1))* will be evaluated to the list *[1,2,3,4]*.

But we do not need those thunks to get laziness. We can simulate them without using any additional feature of the language: an expression *e* can be suspended by means of a partial application of the form $\lambda x.e$ ($x \notin \text{var}(e)$). The type of *x* is not relevant (we can use booleans e.g.). The evaluation of the suspended expression can be forced providing the required argument to the λ -abstraction. The above program could be implemented in this way:

```
type 'a lstL = Nil | Cons of ('a * (bool -> 'a lstL));;

let rec from n = Cons (n, fun true -> from (n+1));;

let rec take n lst = match n, lst with
  | _, Nil -> Nil
  | 0, _ -> Nil
  | _, Cons (x, susp) -> Cons (x, fun true
                                -> take (n-1) (susp true));;

let rec nf l = match l with
  Nil -> []
  | Cons (x, susp) -> x :: nf (susp true);;
```

This last version contains essentially the idea that we will incorporate to our translation, but it presents a problem: while the first version implicitly provides sharing (because OCaml stores the computed result after the evaluation of a thunk, for avoiding reevaluation), the second one does not. In our context this is not only a problem of efficiency, but the intended semantics could be destroyed as we will see in Section 5.2.

5 Our approach to laziness

In our setting we provide laziness annotations for data type constructors. With only this information, the translator will introduce the appropriate suspensions and the corresponding manipulation for them. We start showing a basic translation scheme and then an extension to capture sharing.

5.1 A first translation schema

Consider for example, the program for *from* of Section 4.1, that now can be easily encoded as:

```
:- lazy([_|on]).

from(N) = [N|from(N+1)].

take(0,L) = [] :- !.
take(N,[]) = [] :- !.
take(N,[X|Xs]) = [X|take(N-1,Xs)].

firstNats(M) = take(M,from(1)).
```

The first line declares the Prolog constructor for lists $[-|_]$ to be lazy in the second argument (similar to what is done in the OCaml version). In a preprocessing step, at the level of functional definitions, the translator replaces the (declared) lazy arguments of constructors by λ -abstractions:

```
from(N,L) = [N|fun(_,from(N+1))].

take(0,L) = [] :- !.
take(N,[]) = [] :- !.
take(N,[X|Xs]) = [X|fun(_,take(N-1,Xs@dummy))].

firstNats(M) = take(M,from(1)).
```

The rest of the list in the body of *from* has been replaced by the λ -abstraction $fun(.,from(N+1))$. This abstraction takes a (dummy) anonymous variable as argument and returns the original expression $from(N+1)$. The last rule of *take* contains also a λ -abstraction, but here the situation is a little more complex. The variable *Xs* comes from the second argument of a list in the head, so it is a suspended expression. In order to pass it to *take* in the body first we need to wake it up. This is done by applying a *dummy* argument to it (the constant *dummy* could be replaced by any other term). The general criteria for manipulating suspensions is to introduce them as arguments of constructor symbols that have been declared as lazy (in our example, any tail of a list), and to wake them up when they are used as arguments of functions or constructors that have no such arguments declared as lazy. On the other hand, any variable argument in a constructor, at a position declared as lazy must represent a suspension at run time (in the last rule of *take*, the variable *Xs* is in this situation and it must be woken up when used as an argument for *take* in

the body). Notice that although we are not considering types in our functions, the suspensions obey a discipline of types (as it was the case with the OCaml version).

The next step of the translation will produce the clauses for these rules as explained in Sections 2 and 3, with the corresponding clauses for λ -abstractions and partial applications. With this translation we could now evaluate the goal $firstNats(6,L)$. The resulting answer could be a little surprising:

```
L = [1|fun2(...)]
```

This is correct, as the second argument of lists is a suspended form. In the OCaml version something similar happened and we had to implement a function nf to obtain a normal form (a completely evaluated expression). Our translator incorporates a predefined predicate nf with the same purpose, so we could throw the goal $firstNats(6,L1)$, $nf(L1,L)$ and obtain the expected answer⁶: $L = [1,2,3,4,5,6]$. This scheme works fine, except that it does not consider any sharing, and this can be catastrophic as we will see in the next section.

5.2 Call-time choice and sharing

The previous translation can have an undesirable (or at least unexpected) behaviour when non-determinism is involved. Let us examine an example to illustrate the situation. Consider the classical Prolog generate and test algorithm for sorting a list: generate permutations of the list in a non-deterministic way and test if they are sorted. This algorithm, using (non-deterministic) functions can be easily implemented in Functional Prolog as:

```
permut([]) = [].
permut([X|Xs]) = insert(X,permut(Xs)).

insert(X,[]) = [X].
insert(X,[Y|Ys]) = [X,Y|Ys].
insert(X,[Y|Ys]) = [Y|insert(X,Ys)].

sorted([]) = true.
sorted([X]) = true.
sorted([X,Y|Ys]) = sorted([Y|Ys]) :- X=<Y.
sorted([X,Y|Ys]) = false :- X>Y.

permutSort(Xs) = ifThen(sorted(Ys),Ys) :- Ys=permut(Xs).
```

The function *permut* returns a permutation of the list by inserting the head of such a list into a permutation of the rest of the list; *insert* puts an element into a list at any possible position (by backtraking). The function *sorted* checks if the given list is sorted. And finally, *permutSort* generates a permutation of the list and if it is sorted then returns it (if it is not sorted, by backtraking will search for another permutation). The function *sorted* checks if the given list is sorted. And finally, *permutSort* generates permutations of the list searching for a sorted one

⁶ The predicate nf could be implicitly called in future versions of the translator.

by backtracking. The function *ifThen* is defined in the prelude with the standard meaning.

This algorithm works fine, except for the efficiency. The worst case corresponds to a list in reverse order. For example, the goal *permutSort*([4,3,2,1],L) will produce all the possible permutations of the list before finding the sorted one. This example has become also a classical one in Toy and Curry. Moreover it works much better in Toy and Curry because of laziness. Prolog's and the eager version above must generate permutations completely and then make the test, but with lazy evaluation permutations can be discarded without being fully generated. For example, for sorting the list [4,3,2,1] an eager version will generate all the permutations of the form [3,1,...] while the lazy version can discard all of them as the first two elements (3,1) are not sorted (see [12] for a detailed analysis of this algorithm).

Here, there is not any infinite data structure involved, but laziness is interesting for the sake of efficiency. Then we can use lazy lists with the declaration `:- lazy([_|on]).`, but then a new problem appears. The goal *permutSort*([4,3,2,1],L) will produce six "ordered" lists:

$$[1, 4, 3, 2] [1, 3, 4, 2] \dots [1, 2, 3, 4]$$

that correspond to the element 1 followed by any permutation of [2,3,4]. The problem comes from the two occurrences of the variable *Ys* in the rule

$$\text{permutSort}(Xs) = \text{ifThen}(\text{sorted}(Ys), Ys) \text{ :- } Ys = \text{permut}(Xs).$$

As we are using lazy lists, the condition $Ys = \text{permut}(Xs)$ produces partially evaluated permutations of [4,3,2,1] of the form $[elem|\lambda_.\text{permut}(\text{rest})]$, where *elem* is an element of [4,3,2,1] and *rest* contains the remaining elements. In particular, by backtracking we will find the (partially evaluated permutation) $Ys = [1|\lambda_.\text{permut}([4,3,2])]$. This variable *Ys* is used as an argument in the body of *permutSort* and then the abstraction is duplicated:

$$\text{ifThen}(\text{sorted}([1|\lambda_.\text{permut}([4,3,2]))], [1|\lambda_.\text{permut}([4,3,2]))])$$

This corresponds to a *run-time* choice semantics as both calls can evolve independently. The evaluation of $\text{sorted}([1|\lambda_.\text{permut}([4,3,2]))]$ can be evaluated to *true* as *permut*[4,3,2] can produce (by backtracking) the list [2,3,4]. Then *permutSort* returns the list $[1|\lambda_.\text{permut}([4,3,2])]$, that by normalization can produce the six answers mentioned before.

How can we solve this problem? Or equivalently: how can we implement sharing? The solution uses a technique similar to one implemented in Toy [4] for similar purposes. In our case, it can be implemented extending the λ -abstractions introduced for suspensions with two more arguments (Prolog variables): a flag *Ev* for indicating if the suspension has been evaluated before or not, and the result *Res* of such an evaluation in affirmative case. The new δ -abstractions have the form:

$$\delta \text{ Ev Res } _ . \text{ expr}$$

The dummy argument $_$ has the same meaning as before, *Ev* can be the term *on* or a variable and *Res* can be any Prolog term. The application of a δ -abstraction to the dummy argument is defined as:

Goal	Eager	Lazy funs (all)	Lazy funs (some)	Lazy lists
nPrime(300)	-	0.829399	0.534276	0.182028
nPrime(600)	-	2.65868	1.70961	0.697245
nPrime(1000)	-	overflow	overflow	1.92277
nPrime(6000)	-	overflow	overflow	80.5051
firstPrimes(300)	-	1.06425	0.543337	0.230733
firstPrimes(600)	-	3.58354	1.76382	0.899285
firstPrimes(1000)	-	overflow	overflow	2.19576
firstPrimes(1200)	-	overflow	overflow	3.39396
firstPrimes(1500)	-	overflow	overflow	overflow
permutSort(revLst(10))	8.75339	0.365988	0.160308	0.195278
permutSort(revLst(12))	1000.51	3.18058	1.44293	1.70397
permutSort(revLst(15))	...	96.7763	42.2988	49.6788
permutSort(revLst(16))	...	248.329	110.585	128.547

Fig. 3. Lazy functions versus lazy constructors (times in seconds)

$$@(\delta \text{ Ev Res } _ _ \text{ expr, dummy}) = \begin{cases} \text{Res} & \text{if Ev==on} \\ \text{expr} & \text{otherwise} \end{cases}$$

This idea can be easily implemented in Prolog. δ -abstractions are represented by Prolog terms of the form $\text{funN}(\text{Ev}, \text{Res}, _ , \langle \text{Expr} \rangle)$ (N is a counter for getting different names for the abstractions). These abstractions are flattened as explained in Section 3, but their rules must consider the parameters used for sharing:

$$\text{funN}(\text{Ev}, \text{Res}, \text{Expr}_{\text{Res}}) = H \text{ :- } (\text{Ev} == \text{on}, ! ; \text{Expr}_{\text{Res}}, \text{Ev} = \text{on}), H = \text{Res}.$$

This rule first checks if the expression has been previously evaluated by looking up the flag Ev . In such a case, the resulting value is the one stored in Res . In other case we evaluate Expr , set the flag and return the obtained value. The notation Exp_{Res} indicates that the expression Exp must be flattened in such a way that the resulting value is Res . For example, the function app is defined in the *prelude* as:

```

app([], Xs) = Xs.
app([X|Xs], Ys) = [X|app(Xs, Ys)].
    
```

and it is translated as:

```

app([], Ys, Ys) .
app([X|Xs], Ys, [X|fun1(Ev, Res, Xs, Ys)]).
fun1(Ev, Res, Xs, Ys, Dummy, H) :-
    (Ev==on, !
    ;
    @(Xs, dummy, XsEv), app(XsEv, Ys, R1), H=R1, Ev=on),
    H=Res.
    
```

Now, coming back to our example, the function permutSort works as desired and the goal $\text{permutSort}([4,3,2,1], L)$ will produce a unique answer $L=[1,2,3,4]$. Using lazy lists the algorithm is drastically improved in efficiency as expected. Moreover, the translation incorporates some (quite technical) optimizations that improve the efficiency even more.

6 Examples and performance

In this section we will give an idea of the capabilities of the functional Prolog model proposed in this work and its performance. We are interested in the comparison with similar approaches, in particular, with functional-logic programming and with the approach of [5] that we have summarized in Section 4.1.

The next interesting comparison may be with respect to functional-logic systems, as they also use non-deterministic functions and laziness. Moreover, some implementations as Toy [4] and Packs [10] make a translation into Prolog, what justifies even more the comparison.

It is more difficult to compare our translation with functional languages such as Haskell or OCaml because those programming styles are very different from our proposal, mainly because they are deterministic languages. We have made some tests using examples in which non-deterministic functions are not used, and both Haskell's and OCaml's versions had an improved performance compared with functional Prolog.

All the tests have been done under an Intel Pentium 4 EM64T 3.20 GHz with 1 Gb of RAM memory.

6.1 Lazy functions and lazy constructors

The approach of laziness of Section 4.1 was introduced in [5] as an extension to Ciao Prolog [3]. That proposal is very close to our current approach, as both are functional extensions of Prolog, with optional laziness. It is different from ours in its treatment of higher order and some other aspects, but the most interesting difference is with respect to laziness. Unfortunately, the implementation is not included in the Ciao Prolog distribution and so we must use the translation scheme seen in Section 4.1 for comparison.

As we have pointed out in Section 4.1, implementing this model for laziness is quite straightforward just using the resources of standard Prolog for delaying computations. This approach has the advantage that most of the work about laziness is automatically done by Prolog, using its capabilities for delaying goals. Nevertheless, this model of laziness can be hard to manipulate as, in general, choosing which program functions must be lazily evaluated is not a trivial issue. As an easy solution we could turn every function lazy, but in this case the efficiency of the program would be seriously affected.

Figure 3 shows some experimental measurements using the sieve of Eratosthenes of Section 4.1 and the permutation sort algorithm of Section 5.2 (*revLst(N)* returns a list of length N in reverse order, which is the worst case for this algorithm). The first column contains the goals to solve, the second shows the time measurements for the program without any lazy declaration (*nPrimes* and *firstPrimes* do not terminate), and columns *Lazy funs* (*all* and *some*) contain the running times for the translation of Section 4.1, declaring *all* functions as lazy and only *some* of them as lazy, respectively. For the sieve, the column *some* has been obtained declaring lazy only *from*, *filter* and *sieve* (the minimal laziness needed to ensure termination for the corresponding goal), and for the sorting algorithm only *insert* is declared as lazy. The last column corresponds to the proposed translation, with lazy lists

Input list	Toy	Lazy lists
revLst(10)	0.76	0.28
revLst(12)	6.90	2.35
revLst(12)	6.90	2.35
revLst(12)	62.43	22.97
[fib(22),fib(21),fib(20)]	1.16	1.13
[fib(26),fib(25),fib(24)]	overflow	7.65

Fig. 4. Toy and functional Prolog with lazy lists. Goal: $permutSort(<list>)$ (times in seconds)

declared in the program as `:- lazy([_|on])`. The experiments show that using lazy functions, an appropriate selection of the lazy ones is important to achieve a more efficient program. Using lazy constructors we must not worry about which functions must be declared as lazy, and this version is more efficient in time and space for the sieve. In the case of sorting, the performance of the version with *some* lazy functions is similar to the one for lazy constructors.

Functional-logic languages like Toy and Curry are also close to the proposal of this paper, as they use lazy non-deterministic functions and logic variables. Furthermore, these languages are usually implemented by a compilation into Prolog code. We have tested the $permutSort$ algorithm presented in Section 5.2 in our functional Prolog translation and Toy (with minor syntactic changes). Figure 4 shows that the performance is improved significantly in the functional Prolog version.

The second example of Figure 4 is more interesting as the differences come directly from the lazy models that we are using. The function $fib(N)$ evaluates the N -th number of the Fibonacci series. It is implemented with exponential complexity ($fib(0)=0$; $fib(1)=1$; $fib(N)=fib(N-1)+fib(N-2)$). A goal like

$$permutSort([fib(22),fib(21),fib(20)])$$

has an undesired behaviour in Toy as it needs to reevaluate each item of the list several times, as the value is not reused when trying another permutation for sorting the list. This is not the case with our model of laziness, as only lists are lazily evaluated. The calls to fib are evaluated only once and then the sorting algorithm proceeds to sort the list.

6.2 Mixing lazy and eager evaluation

Having lazy annotations for constructors allows to get a finer control on the evaluation and achieve a better performance in some situations. We illustrate this topic with Pascal's triangle in the next example. Such a triangle is an infinite structure of the form:

$$\begin{array}{rcl}
 1 & & \text{row } 0 \\
 1 & 1 & \text{row } 1 \\
 1 & 2 & 1 & \text{row } 2 \\
 1 & 3 & 3 & 1 & \text{row } 3 \\
 1 & 4 & 6 & 4 & 1 & \text{row } 4 \\
 & \dots & & & &
 \end{array}$$

Each row can be represented as a list, hence being the triangle an infinite list of lists. In general, the row i -nth ($i > 0$) is obtained adding pair of consecutive elements of the previous row and putting 1 as the first and last element in the resulting row. Given a row F , say $[1, 3, 3, 1]$ we can elegantly obtain the next row as follows: we build the lists $[0, 1, 3, 3, 1]$ and $[1, 3, 3, 1, 0]$ and then add the elements at the same positions obtaining the list $[1, 4, 6, 4, 1]$. This process can be done within a single (functional) expression:

```
zipWith(+), [0|F], app(F, [0])).
```

The function *zipWith* is defined as:

```
zipWith(F, [], Xs) = [] :-!.
zipWith(F, Xs, []) = [] :-!.
zipWith(F, [X|Xs], [Y|Ys]) = [(F@X)@Y|zipWith(F, Xs, Ys)].
```

and *app* stands for the concatenation of lists.

With these ideas, a function *pas* for evaluating the triangle of Pascal can be written as:

```
:-include(prelude).

:- lazy([_|on]).

iter(F) = [F|iter(zipWith(+), [0|F], app(F, [0])))].
pas = iter([1]).

% testing functions
row(N) = nth(N, pas).
comb(N, M) = nth(M, (nth(N, pas))).
```

The function *iter(F)* evaluates all the next rows (an infinite number) from a given row F and *pas* calls it with the initial row $[1]$ to obtain the complete triangle. The functions *row* and *comb* are functions for testing the program (*nth(N)* is defined in the *prelude* and returns the N -th element of a list, starting at position 0). *row(N)* returns the N -th row of the triangle and *comb(N, M)* returns the combinatorial number $\binom{N}{M}$ using the fact that the M -th element of the N -th row of the triangle is exactly that combinatorial number. We can obtain for example the row 8 of the triangle by throwing the goal *row(8, K)*, *nf(K, L)*:

```
L = [1, 8, 28, 56, 70, 56, 28, 8, 1]
```

or we can obtain the combinatorial number $\binom{18}{5}$ with *comb(18, 5, N)* getting $N=8568$.

Notice that due to the lazy evaluation of lists, the first goal only evaluates the first 8 rows, and the second one evaluates only the initial prefix of the first 18 rows (for example it does not need to evaluate $\binom{16}{14}$).

Having laziness associated to data type constructors instead of functions provides a finer control over the construction of data structures. In this example, we use (lazy) lists both for representing the rows of the triangle and also to collect those rows, but one could think about these lists as different kinds of structures. In particular, if we were interested in getting complete rows of the triangle (first goal

Goal	All lazy	Lazy/eager
row(400)	0.691386	0.248232
row(600)	1.77859	0.560645
row(800)	overflow	1.01706
row(2000)	overflow	6.71622
row(4000)	overflow	30.0093
comb(500, 250)	0.87144	0.561656
comb(800, 400)	2.13438	2.18744
comb(1000, 500)	5.29124	4.3969

Fig. 5. Mixing lazy and eager evaluation

above), we could improve this code by distinguishing two data type structures. Rows could be represented as (a new kind of) lists with constructors *nil* and *cons*:

$$row ::= nil \mid cons(e, row)$$

These *rows* would be strictly evaluated, and we could use lazy lists as we did before for collecting the *rows* of the triangle as follows:

```
:- include( Prelude ).

:- lazy( [ _ | on ] ).

iter2(F) = [ F | iter2( zipWithRows( (+), cons( 0, F ),
                                appendRows( F, cons( 0, nil ) ) ) ) ].

pas2 = iter2( cons( 1, nil ) ).
row2(N) = nth( N, pas2 ).
```

where *zipWithRows* and *appendRows* are the versions for *zipWith* and *app*, but for the *rows* we are using. This program is faster than the previous one, specially for getting an entire row of the triangle (see Figure 5).

6.3 Metaprogramming

In practice, it is very usual to use meta-predicates when programming in Prolog. In this section we show how our functional extension can be useful combined with meta-programming. We explore the classic *count-down* problem: we must find how to combine a list of operands (integers) by means of arithmetical operations to obtain a given total. This problem can be solved by a *generate-and-test* algorithm: generate an expression in a non-deterministic way and test if it is a solution, i.e., if its evaluation gives the required total.

Figure 6 shows the program for solving this problem. An expression can be either a number or an operator applied to two expressions. We use several (non-deterministic) functions for generating expressions: *oneOf(L)* returns an element of the list *L*; *split(L)* divides the list *L* into two sublists; and *genExp(Ns)* uses these functions to build an expression with the numbers of *Ns*.

The term decomposition predicate *=.* combined with the function *oneOf* allows for an elegant definition of *genExp(Ns)*.

The evaluation function *eval* is defined in a natural way using Prolog cuts (the

```

% expression ::= number | plus exp exp | minus exp exp
                | mult exp | div exp exp

% non deterministic generation of expressions
genExp(Ns) = oneOf(Ns).
genExp(Ns) = E :-
    split(Ns)=(As,Bs),
    E=..[oneOf([plus,minus,mult,div]),genExp(As),genExp(Bs)].

oneOf([N|Ns]) = N.
oneOf([N|Ns]) = oneOf(Ns).

split([X,Y]) = ([X],[Y]).
split([X,Y]) = ([Y],[X]).
split([N|Ns]) = S :- split(Ns)=(As,Bs),
                    (S=([N|As],Bs); S=(As,[N|Bs])).

% evaluation of expressions
eval(plus(E1,E2)) = eval(E1)+eval(E2) :- !.
eval(minus(E1,E2)) = eval(E1)-eval(E2) :- !.
eval(mult(E1,E2)) = eval(E1)*eval(E2) :- !.
eval(div(E1,E2)) = V1/V2 :-
    !, V1=eval(E1), V2=eval(E2),
    V2\==0, V1>V2, mod(V1,V2)==0.
eval(N)=N.

% solution for the count-down problem
sol(V,L)=E :- E=genExp(L), V==eval(E).

```

Fig. 6. Arithmetical expressions: generation and evaluation

last rule stands for a single number). The rule for division must make some tests as it is understood as integer division. Finally, the function $sol(V,L)$ generates an expression with the operands of the list L and tests if it is a solution. By backtracking this function will find the solution for the problem if it exists. For example, $sol(28,[3,6,4,5])$ will return $mult(plus(5, div(6, 3)), 4)$, that is, $(5+6/3)*4$.

If we evaluate $sol(26,[4,5,3])$ we do not obtain any solution as the numbers 4, 5 and 3 do not allow to get the exact value 26. It would be interesting to obtain the *best approximation* to the solution from the program above. A typical solution in Prolog for this kind of problems is to use some meta-predicates to explore the search space. Using higher order functions we can encapsulate this search operation. It would be nice to have a function

$$best(Gen,Data,Comp)$$

that returns the best alternative given: a non-deterministic function Gen that generates the search space using the arguments provided in $Data$, where the best solution depends on a comparison criteria $Comp$. This function would allow to get the best approximation to a value V given a list of numbers L as:

```
bestApprox(V,L) = best(genExpVal,[V,L],better).
```

```
genExpVal(V,L) = (E,abs(V-eval(E))) :- genExp(L)=E.
```

```
better((E1,V1),(E2,V2)) = (E1,V1) :- V1<V2, !.
```

```
better((E1,V1),(E2,V2)) = (E2,V2).
```

Here the function $genExpVal(V,L)$ returns a *candidate* to solution: a pair with an expression built-up with the numbers of L and the *valorization* (not evaluation) of that expression according to the problem. In this problem the valorization is the difference between the value we are searching for V and the result of evaluating the expression. For example, $genExpVal(17,[3,4,1])$ may return $(3*4+1,4)$ as $3*4+1=13$ and the difference with the total $17-13=4$; another result may be $(3*4,5)$ as $17-12=5$. The function $better(C1,C2)$ returns the best between the candidates $C1$ and $C2$, where a candidate is a pair expression-valorization as we have seen. Now, for the expression $bestApprox(26,[4,5,3])$ we would obtain the expression $mult(3, plus(5, 4))$ and the valorization 1 ($3*(5+4)=27$ and $27-26=1$).

The only open question is how to define the function $best(Gen,Data,Comp)$. Figure 7 shows the code for this function. It initiates the search producing a candidate and asserting it as the best solution for the moment. Then, $bestAux$ generates all the possible candidates updating the best solution found at each step.

The important fact is that this function $best$ can be used to solve a variety of problems of optimization. Other functions for exploring the search space can be appropriately parametrized and encoded, analogously to the encapsulated search primitives of Curry [2].

```
best(F,Args,Better) = R :-
    applyLst(F,Args)=EV, retractall(currentBest(_)),
    assert(currentBest(EV)), !, bestAux(F,Args,Better)=R.

bestAux(F,Args,Better)= R :-
    applyLst(F,Args)=E, currentBest(C),
    applyLst(Better,[E,C])=E, retractall(currentBest(_)),
    assert(currentBest(E)), fail.
bestAux(_,_,_) = R :- !, currentBest(R).

applyLst(F,[]) = F.
applyLst(F,[X|Xs]) = applyLst(F@X,Xs).
```

Fig. 7. Encapsulated search for the best candidate

7 Conclusions and future work

In this work we show how to extend Prolog with functions performing a translation of function rules into Prolog predicates. We introduce a simple syntax for function definition and a transformation of higher order to first order syntax including lambda abstractions. With these features we can talk about *functional Prolog*. The extended language implements an eager evaluation mechanism, but we explore some

alternatives for introducing lazy evaluation by means of user annotations.

It is possible to manipulate optional laziness by introducing annotations on functions or on constructors. In practice it is easier to use the second alternative because to decide which functions in a program must be lazily evaluated may be a hard task, while it is not difficult to think about the data structures that must be lazily constructed. Moreover, experimental measurements show a better performance when using lazy constructors.

The manipulation of laziness at the level of constructors can be done by introducing artificial lambda abstractions that suspend the evaluation of the desired arguments. But this approach presents two problems: for obtaining *call-time* choice semantics some sharing mechanism is needed, and the explicit manipulation of lazy constructor symbols is hard in practice and complicates the writing/reading of programs.

For the first problem we extend lambda abstractions with a sharing structure that ensures *call-time* choice. For the second one, the translation incorporates a complete mechanism for delaying computations and forcing later on their evaluation. Similar transformations have been proposed for eager functional languages [6] and are close to *context-sensitive rewriting* [7,14]. As a future work it would be interesting to study the relation of our lazy annotations with context-sensitive rewriting, at least for the declarative fragment of functional Prolog (excluding metaprogramming).

We have presented a collection of examples showing the capabilities of the proposal. In particular, we obtain a fine control on laziness that can be used to improve the efficiency of programs. On the other hand, combining the meta-predicates provided by Prolog with the functional extension we obtain sophisticated resources for exploring the search space.

The current translator is a Prolog file of about 500 lines of code. Some optimizations and new features can be added as future work. In particular, it would be interesting to improve the interface with the user.

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