Quantitative Logic Programming Revisited

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Introduction (I)

Motivation: work out an expressive framework for *Constraint Funcional-Logic Programming* with uncertainty.

First step: revise an early proposal by *van Emden* (1986): *Quantitative Logic Programming* and improve it in two ways:

- generalizing *van Emden*'s *QLP* to a generic scheme $QLP(\mathcal{D})$ parametrized by a qualification domain $\mathcal{D}$.
- generalizing *van Emden*'s results by providing stronger ones, both in declarative semantics and goal solving.

In addition, our $QLP(\mathcal{D})$ scheme has interesting instances that have been implemented using *CLP*.

Introduction (and II): *Van Emden*'s proposal

- *Van Emden* considered *Quantitative Logic Programs* (*QLP*) as sets of clauses of the form:

$$A \leftarrow f - B_1, \ldots, B_n$$

  for which the certainty being propagated to a clause head was $f \times b$ where:

  f - *attenuation factor* with $f \in (0, 1]$

  b - minimum of the certainty factors known for the body atoms

- This approach led to:
  - general results on model theoretic and fixpoint semantics (similar to those for classical LP)
  - procedure for computing the certainty factors in the least Herbrand model of a given program by an alpha-beta heruristic (only for ground atoms with finite search trees)

Qualification Domains (I): Axioms

- Structure $\mathcal{D} = \langle D, \sqsubseteq, \bot, \top, \circ \rangle$ such that:
  - $\langle D, \sqsubseteq, \bot, \top \rangle$ is a lattice with extreme points $\bot$ and $\top$ w.r.t. $\sqsubseteq$.
  - $\circ : D \times D \rightarrow D$, called *attenuation operation*, verifies:

  (a) $\circ$ is associative, commutative and monotonic w.r.t. $\sqsubseteq$.

  (b) $\forall d \in D : d \circ \top = d$.

  (c) $\forall d \in D : d \circ \bot = \bot$.

  (d) $\forall d, e \in D \setminus \{\bot, \top\} : d \circ e \sqsubset e$.

  (e) $\forall d, e_1, e_2 \in D : d \circ (e_1 \sqcap e_2) = d \circ e_1 \sqcap d \circ e_2$.

- We generalize *van Emden*'s *QLP* to *QLP*$(\mathcal{D})$ where:
  - qualification values $d \in D \setminus \{\bot\}$ instead of $d \in (0, 1]$.
  - the *glb* operator $\bigsqcap$ instead of `min` .
  - $\circ$ instead of $\times$ (product).

Qualification Domains (and II): Instances

- $\mathcal{B} = (\{0, 1\}, \leq, 0, 1, \wedge)$ (Classical Boolean Values).
- $\mathcal{U} = ([0, 1], \leq, 0, 1, \times)$ (*van Emden*'s Certainty Degrees).
- $\mathcal{W} = ([0, \infty], \geq, \infty, 0, +)$ (Proof trees' depths).

- The cartesian product $\mathcal{D} = \mathcal{D}_1 \times \mathcal{D}_2$ of two given qualification domains is always another qualification domain.
- Note: $\mathcal{U} \times \mathcal{W}$ qualifies both certainties and proof trees' depths.

Syntax (I): Programs

Program: set of *qualified definite Horn clauses* of the form

$$A \leftarrow d - B_1, \ldots, B_k$$

Example 1a

```
cruel(X) <-0.90- human(X), eats(X,Y), animal(Y)
```

Example 1b

```
cruel(X) <-1- human(X), eats(X,Y), animal(Y)
```

Example 1c

```
cruel(X) <-0.90,1- human(X), eats(X,Y), animal(Y)
```

Syntax (and II): Interpretations and Models

- *$\mathcal{D}$-annotated atom*: $A \sharp d$.
- *Open $\mathcal{D}$-annotated atom*: $A \sharp W$ (W is intended to take values over $D \setminus \{\bot\}$).

$\mathcal{D}$-entailment relation: is defined as $A \sharp d \succcurlyeq_{\mathcal{D}} A' \sharp d'$ iff there is some substitution θ such that $A' = A\theta$ and $d' \sqsubseteq d$.

Open Herbrand interpretation over $\mathcal{D}$: is defined as any set $\mathcal{I}$ of $\mathcal{D}$-annotated atoms which is closed under $\mathcal{D}$-entailment. The set of all interpretations over $\mathcal{D}$ is a complete lattice under $\subseteq$.

Model: An interpretation $\mathcal{I}$ is a *model* of a clause $A \leftarrow d - B_1, \ldots, B_k$ in the program $\mathcal{P}$ iff for any θ and any $d_1, \ldots, d_k \in D \setminus \{\bot\}$ such that $B_i\theta \sharp d_i \in \mathcal{I}$ ($1 \leq i \leq k$), one has $A\theta \sharp (d \circ \prod\{d_1, \ldots, d_k\}) \in \mathcal{I}$. And we say $\mathcal{I}$ is a model of $\mathcal{P}$ if $\mathcal{I}$ is a model of each clause in $\mathcal{P}$.

Declarative Semantics (I)

Qualified Horn Logic over $\mathcal{D}$: $QHL(\mathcal{D})$ is defined as a deductive system consisting just of one inference rule $QMP(\mathcal{D})$ called *Qualified Modus Ponens* over $\mathcal{D}$. If there are some $(A \leftarrow d - B_1, \ldots, B_k) \in \mathcal{P}$, some substitution θ such that $A' = A\theta$ and $B_i' = B_i\theta$ for all $1 \leq i \leq k$ and $d' \sqsubseteq d \circ \bigsqcap\{d_1, \ldots, d_k\}$, the following inference step is allowed:

$$\frac{B_1' \sharp d_1 \quad \cdots \quad B_k' \sharp d_k}{A' \sharp d'} \quad \mathrm{QMP}(\mathcal{D})$$

Notation $\mathcal{P} \vdash_{\mathrm{QHL}(\mathcal{D})} A \sharp d$ or $\mathcal{P} \vdash^{n}_{\mathrm{QHL}(\mathcal{D})} A \sharp d$.

Declarative Semantics (II)

Example 2

```
cruel(X) <-0.90- human(X), eats(X,Y), animal(Y)
animal(bird) <-1.0-
human(eve) <-1.0-
human(mother(X)) <-0.90- human(X)
eats(eve,X) <-0.30- animal(X)
eats(mother(X),Y) <-0.70- eats(X,Y)
```

Declarative Semantics (II)

Example 2

```
cruel(X) <-0.90- human(X), eats(X,Y), animal(Y)
animal(bird) <-1.0-
human(eve) <-1.0-
human(mother(X)) <-0.90- human(X)
eats(eve,X) <-0.30- animal(X)
eats(mother(X),Y) <-0.70- eats(X,Y)
```

```
                                          ________________
                                          animal(bird)#1.0
   ______________                    ______________________
   human(eve)#1.0                    eats(eve,bird)#0.30                ________________
-------------------------   -------------------------------   ----------------
human(mother(eve))#0.90     eats(mother(eve),bird)#0.21       animal(bird)#1.0
--------------------------------------------------------------------------------
                         cruel(mother(eve))#0.15
```

Declarative Semantics (II)

Example 2

```
cruel(X) <-0.90- human(X), eats(X,Y), animal(Y)
animal(bird) <-1.0-
human(eve) <-1.0-
human(mother(X)) <-0.90- human(X)
eats(eve,X) <-0.30- animal(X)
eats(mother(X),Y) <-0.70- eats(X,Y)
```

$$\dfrac{\dfrac{\dfrac{}{\texttt{human(eve)\#1.0}}}{\texttt{human(mother(eve))\#0.90}} \quad \dfrac{\dfrac{\dfrac{}{\texttt{animal(bird)\#1.0}}}{\texttt{eats(eve,bird)\#0.30}}}{\texttt{eats(mother(eve),bird)\#0.21}} \quad \dfrac{}{\texttt{animal(bird)\#1.0}}}{\texttt{cruel(mother(eve))\#0.15}} *$$

(*) $0.15 \leq 0.90 * \min\{0.90, 0.21, 1.0\} = 0.189$.

Declarative Semantics (and III)

Characterizations of the least Herbrand model:

1. The least fixpoint $\mu(\mathrm{T}_\mathcal{P})$ is the least Herbrand model of $\mathcal{P}$, noted as $\mathcal{M}_\mathcal{P}$.
2. $\mathcal{M}_\mathcal{P} = \{A \sharp d \mid \mathcal{P} \vdash_{\mathrm{QHL}(\mathcal{D})} A \sharp d\}$.

Goals

- Initial goals:

$$\cdots, A_i \sharp W_i, \cdots \ [\!] \ \epsilon \ [\!] \ \cdots, W_i \sqsupseteq \beta_i, \cdots$$

- General goals:

$$\cdots, A_i \sharp W_i, \cdots \ [\!] \ \sigma \ [\!] \ \cdots, W' = d \circ \bigsqcap\{W'_1, \ldots, W'_k\},$$

$$\cdots, \alpha \circ W_i \sqsupseteq \beta_i, \cdots$$

- Solved goals:

$$[\!] \ \sigma \ [\!] \ \cdots, W' = d \circ \bigsqcap\{W'_1, \ldots, W'_k\}, \cdots$$

SLD($\mathcal{D}$) Resolution (I)

- Resolution step:

$$\cdots, A \sharp W, \cdots \ \Box\ \sigma\ \Box\ \cdots, \alpha \circ W \sqsupseteq \beta, \cdots \Vdash_{C_1,\sigma_1}$$

$$\begin{aligned}(\cdots, B_1 \sharp W_1, \cdots, B_k \sharp W_k, \cdots)\sigma_1 \ \Box\ \sigma\sigma_1\ \Box\ \cdots, \\ d \circ \alpha \circ W_1 \sqsupseteq \beta, \cdots, d \circ \alpha \circ W_k \sqsupseteq \beta, \cdots, \\ W = d \circ \textstyle\bigsqcap\{W_1, \ldots, W_k\}, \cdots\end{aligned}$$

where $C_1 \equiv (H \leftarrow d - B_1, \ldots, B_k) \in \mathcal{P}$ such that $d \circ \alpha \sqsupseteq \beta$ and σ_1 the m.g.u. between A and H.

- Resolution computations:

$$G_0 \Vdash_{\sigma_1} G_1 \Vdash_{\sigma_2} \cdots \Vdash_{\sigma_n} G_n \equiv \sigma_1\sigma_2\cdots\sigma_n \ \Box\ \Delta_n$$

with G_0 initial and G_n solved, and the computed answer (σ, μ) where $\sigma = \sigma_1\sigma_2\cdots\sigma_n$ and $\mu(W)$ computed according to Δ_n.

SLD($\mathcal{D}$) Resolution (and II)

Example 3

`eats(X,Y)#W | {} | W >= 0.20` $\Vdash_{\texttt{eats.2},\{\texttt{X}\mapsto\texttt{mother(X')}\}}$

SLD($\mathcal{D}$) Resolution (and II)

Example 3

```
eats(X,Y)#W | {} | W >= 0.20
eats(X',Y)#W1 | {X ↦ mother(X')} |
   W = 0.70 * min {W1},
   0.70 * W1 >= 0.20
```

$\Vdash_{\texttt{eats.2},\{\texttt{X}\mapsto\texttt{mother(X')}\}}$

$\Vdash_{\texttt{eats.1},\{\texttt{X'}\mapsto\texttt{eve}\}}$

SLD($\mathcal{D}$) Resolution (and II)

Example 3

eats(X,Y)#W | {} | W >= 0.20 $\Vdash_{\texttt{eats.2},\{\texttt{X}\mapsto\texttt{mother(X')}\}}$

eats(X',Y)#W1 | {X ↦ mother(X')} |
W = 0.70 * min {W1},
0.70 * W1 >= 0.20 $\Vdash_{\texttt{eats.1},\{\texttt{X'}\mapsto\texttt{eve}\}}$

animal(Y)#W2 | {X ↦ mother(eve)} |
W = 0.70 * min {W1},
W1 = 0.30 * min {W2},
0.30 * 0.70 * W2 >= 0.20 $\Vdash_{\texttt{animal},\{\texttt{Y}\mapsto\texttt{bird}\}}$

SLD($\mathcal{D}$) Resolution (and II)

Example 3

```
eats(X,Y)#W | {} | W >= 0.20                      ||-eats.2,{X↦mother(X')}
eats(X',Y)#W1 | {X ↦ mother(X')} |
    W = 0.70 * min {W1},
    0.70 * W1 >= 0.20                             ||-eats.1,{X'↦eve}
animal(Y)#W2 | {X ↦ mother(eve)} |
    W = 0.70 * min {W1},
    W1 = 0.30 * min {W2},
    0.30 * 0.70 * W2 >= 0.20                      ||-animal,{Y↦bird}
| {X ↦ mother(eve), Y ↦ bird} |
    W = 0.70 * min {W1},
    W1 = 0.30 * min {W2},
    W2 = 1.0
```

SLD($\mathcal{D}$) Resolution (and II)

Example 3

```
eats(X,Y)#W | {} | W >= 0.20                            ||-eats.2,{X↦mother(X')}
eats(X',Y)#W1 | {X ↦ mother(X')} |
   W = 0.70 * min {W1},
   0.70 * W1 >= 0.20                                     ||-eats.1,{X'↦eve}
animal(Y)#W2 | {X ↦ mother(eve)} |
   W = 0.70 * min {W1},
   W1 = 0.30 * min {W2},
   0.30 * 0.70 * W2 >= 0.20                              ||-animal,{Y↦bird}
| {X ↦ mother(eve), Y ↦ bird} |
   W = 0.70 * min {W1},
   W1 = 0.30 * min {W2},
   W2 = 1.0
```

Computed answer: $\{X \mapsto \texttt{mother(eve)}, Y \mapsto \texttt{bird}\} \,[]\, \{W \mapsto \texttt{0.21}\}$

Properties of SLD($\mathcal{D}$) Resolution

- Definition of Solution: A pair os substitutions (θ, ρ) is called solution of a goal $G \equiv \overline{A} \parallel \sigma \parallel \Delta$ iff:
  (i) $\theta = \sigma\theta$.
  (ii) $\rho \in Sol(\Delta)$.
  (iii) $\mathcal{P} \vdash_{\mathrm{QHL}(\mathcal{D})} A\theta \sharp W\rho$ for every annotated atom in $\overline{A}$.

Soundness: Assume $G_0 \Vdash^* G$ and $G = \sigma \parallel \Delta$ solved. Let (σ, μ) be the computed answer to G. Then (σ, μ) is a solution of G_0.

Strong Completeness: Assume a given solution (θ, ρ) for G_0 and any fixed strategy for choosing the selected atom at each resolution step. Then there is some computed answer (σ, μ) for G_0 which is a more general solution than (θ, ρ).

Implementation: translating to $\mathcal{TOY}$

Requirement *CLP* or *CFLP* system with support for $\mathcal{C}_\mathcal{D}$ constraints.

$$C \equiv a(\overline{t}) \leftarrow d - b_1(\overline{s_1}), \ldots, b_k(\overline{s_k})$$

$$\begin{array}{rcl} C^t \equiv a(\overline{t}, Alpha, W, Beta) & \leftarrow & d \circ Alpha \sqsupseteq Beta, \\ & & b_1(\overline{s_1}, d \circ Alpha, W_1, Beta), \\ & & \vdots \\ & & b_k(\overline{s_k}, d \circ Alpha, W_k, Beta), \\ & & W = d \circ \bigsqcap\{W_1, \ldots, W_k\} \end{array}$$

$$G \equiv a_1(\overline{t_1}) \sharp W_1, \ldots, a_m(\overline{t_m}) \sharp W_m \; [] \; W_1 \sqsupseteq \beta_1, \ldots, W_m \sqsupseteq \beta_m$$

$$G^t \equiv a_1(\overline{t_1}, \top, W_1, \beta_1), \ldots, a_m(\overline{t_m}, \top, W_m, \beta_m)$$

Conclusions

- Generalization of the QLP proposal by *van Emden* to a generic scheme $QLP(\mathcal{D})$ covering uncertainty and more.
- Sound and complete goal solving procedure improving *van Emden*'s QLP.
- Implementation of some $QLP(\mathcal{D})$ instances by means of a translation to $CLP(\mathcal{C}_{\mathcal{D}})$ realized in $\mathcal{TOY}$.

Future work:

- Simple treatment of negation.
- Threshold constraints in clause bodies.
- Similarity-based reasoning.
- Extension with lazy functions and constraints.